Lumped-parameter models as a tool for determining the hydrological parameters of some groundwater systems based on isotope data

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Abstract The use of lumped-parameter models for interpreting environmental tracer data is based on the assumption that the transit time distribution function of the tracer particles through the groundwater system under consideration is known or can be assumed. These functions describe the whole spectrum of the transit times of single tracer particles transported through the system between the entrance (recharge area) and the exit (spring, stream, pumping well). The main parameter of each model is the mean transit time of water through the system. This parameter is determined by calibrating the model using the environmental tracer concentrations measured in the output. Additional useful hydrological information can be derived from the model parameters. The paper presents the transit time distribution functions of some commonly used lumped-parameter models and shows examples of their application for solving bank filtration problems (drinking water supply for Passau, Germany), and determining the water dynamics and water balance in a heterogeneous karstic aquifer (Schneealpe, Austria).

INTRODUCTION

Recent progress in numerical methods and multilevel samplers focused the attention of modellers on two- or three-dimensional solutions to the transport (dispersion) equation. To solve the problem of pollutant transport, numerical models are generally used which are based on the finite-element or the finite-difference methods. However, to apply them in practice, an extensive database is required. These data are very often not available, and when an extensive database is available, data are usually missing at the beginning of most investigations. Lumped-parameter models require only a minimum of information concerning the turnover zone of the groundwater and the record of non-reactive tracer concentrations measured in the input and the output from the system as a function of time. These models can be simply validated with the available hydrogeological data. It should be noted that if the lumped-parameter model is already calibrated for the system under consideration, then this model can also be used to evaluate other problems e.g. the effects of surface contamination on the underground water.

The environmental isotopes e.g. tritium and stable isotopes \(^{18}\text{O}\) or deuterium) are suitable for tracing the origin of water in the hydrological cycle because they are constituents of the water molecule. Therefore, the use of environmental isotope techniques may, in addition to conventional hydrological methods, provide further insight into, for instance, determining the origin of water, the storage properties of a
catchment; the water dynamics in a groundwater system or the reaction between surface and groundwater by bank filtration. The quantitative evaluation of these data is often based on using lumped-parameter models, which were developed for chemical engineering and adopted by isotope hydrology.

**BASIC CONCEPT OF LUMPED-PARAMETER MODELS**

The principles and use of lumped-parameter models, also called black box models, are well described e.g. by Maloszewski & Zuber (1982, 1993, 1996). Each model is characterized by the transit time distribution function of tracer particles transported between the input (recharge area) and the output (discharge area). This function has to be known (assumed) based on hydrological knowledge of the system under consideration. In most cases the transit time distribution functions have one or two unknown (fitting) parameters which can be determined by calibrating the model to the experimental data observed in the outflows from the groundwater system. The input concentration of the environmental tracer is directly measured or has to be calculated from the precipitation amount, infiltration coefficients and tracer content in precipitation (Maloszewski & Zuber, 1982; Grabczak et al., 1984; Stichler et al., 1986).

The main parameter of all models is the mean transit time of water through the system ($T$), which is related to the water volume in the system ($V$) and the volumetric flow rate through the system ($Q$) according to the following relationship:

$$V = QT$$

The mean transit time represents the average of the number of flow times at the individual streamlines in the aquifer, weighted with the amount of water flowing.

![Diagram of conceptual models](image-url)

Fig. 1 The schematic presentation of the most frequently used conceptual models of the groundwater system (after Maloszewski et al., 1983).
Generally, it is assumed that the groundwater system can be considered as a closed system, sufficiently homogeneous, being in the steady state, having a defined input (recharge or infiltration area) and a corresponding output in the form of pumping wells, springs or streams draining the system. However, the conceptual model must not be limited to the “one-box” representation of the system. The conceptual model can also be further developed as was shown in e.g. Maloszewski et al. (1983) and within this paper. The schematic presentation of the most frequently used conceptual models of the groundwater system is shown in Fig. 1.

To determine the mean transit time of water, the temporal variation of the measured tracer input concentration, $C_{in}(t)$, is used to calculate the tracer output concentration, $C_{out}(t)$, which, in turn, is compared with the concentrations measured in the output from the system. The relationship between input and output concentrations is described by the well known convolution integral:

$$C_{out}(t) = \int_{0}^{\infty} C_{in}(t-\tau) g(\tau) \exp(-\lambda \tau) d\tau$$

(2)

with $\lambda$ being the decay constant of radioactive tracers.

The most common kinds of transit time distribution functions, $g(\tau)$, which automatically define the lumped-parameter model (3), are:

**Piston-flow model**

$$g(\tau) = \delta(\tau - T)$$

(3.1)

**Exponential model**

$$g(\tau) = \frac{1}{T} \exp\left(-\frac{\tau}{T}\right)$$

(3.2)

**Dispersion model**

$$g(\tau) = \frac{1}{\sqrt{4\pi P_D T}} \frac{1}{\tau} \exp\left[-\frac{(1-\tau/T)^2}{4P_D T/T}\right]$$

(3.3)

where $P_D$ is the dispersion parameter.

Each model has one or two unknown (fitting) parameters which can be found by solving equation (2) with the assumed lumped-parameter model (3). To solve the inverse problem, e.g. to find model parameters, the user friendly software “FLOWPC” described in detail in Maloszewski & Zuber (1996) can be applied. The selection of the model depends on the hydrogeological conditions in the groundwater system under consideration. Generally, the dispersion and piston flow models are applicable for confined or partially confined aquifers, while the exponential model can be used only for unconfined aquifers. A detailed description of the model applicability can be find in Maloszewski & Zuber (1982, 1996) or Zuber (1986).

**EXAMPLES OF MODEL APPLICATION**

A number of case studies have been presented and well reviewed, e.g. Maloszewski (1994), Maloszewski & Zuber (1982, 1993, 1996) and Zuber (1986). Only two selected examples of lumped-parameter model application are presented below.
River bank filtration

In river bank filtration studies, advantage is taken of the seasonal variations in the stable isotope composition of river water, and of the difference between its mean value and the mean value of local groundwater, e.g. Hötzl et al. (1989), Maloszewski et al. (1990) and Stichler et al. (1986). Measurements of the $^{18}$O-content in river water, local groundwater and pumping wells were used to determine the fraction and the flow times of the Danube bank filtrate. The study was performed on a small island called Soldatenau (0.3 km²), where the drinking water supply has been constructed for the city of Passau in Germany (Stichler et al., 1986). The infiltrating Danube water was mixed with the local groundwater, which under natural flow conditions is drained by the river. In Fig. 2 a schematic presentation of the flow pattern and its lumped-parameter model is shown. The portion of the river water ($p$) was calculated from the mean tracer content of all components ($C$) applying the following formula:

$$p = \frac{C_{PW} - C_{LG}}{C_{DR} - C_{LG}}$$  (4)

where subscripts: $PW$, $LG$, and $DR$ are for pumping well, local groundwater and Danube River water, respectively.

Due to the fact that the local groundwater had no seasonal variation, $C_{LG}(t) \approx$ constant, the following formula is adequate for the situation shown in Fig. 2:

$$C_{PW}(t) = p \int_{0}^{\infty} C_{DR}(t - \tau) g(\tau) d\tau + (1 - p) C_{LG}$$  (5)

where the input function $C_{DR}(t)$ was taken as the weighted monthly means of the $^{18}$O-content in Danube River water for a period of 36 months.

Using equation (4) the portion of river water in the pumping well was calculated to be $p = 0.80$. Due to the hydrogeological conditions the dispersion model (3.3) was applied and yielded the best fit (Fig. 3) with mean transit time of water $T = 60$ days and $P_D = 0.12$ (corresponding to $c_L = 25$ m). This calibrated model was later used to predict the response of the pumping well to a hypothetical pollutant in the river water.

Water dynamic in a karstic aquifer

The Schneealpe karst massif of Triassic limestones and dolomites (altitude up to 1800 m a.s.l.) is situated 100 km southwest of Vienna in Kalkalpen, and is the main drinking water resource for the city (Rank et al., 1992). The catchment area of about
Fig. 3 Observed (triangles) and fitted with dispersion model (solid line) $^{18}$O output concentrations in the pumping well PSI ($T = 60$ days, $P_D = 0.12, p = 0.8$) at the island Soldatenau (after Stichler et al., 1986).

$23 \text{ km}^2$ is drained by two springs: the Wasseralmquelle ($2001 \text{s}^{-1}$) and the Siebenquellen ($3101 \text{s}^{-1}$). This karstic aquifer is approximated by two interconnected parallel flow systems of a fissured-porous aquifer and karstic channels. The fissured-porous aquifer with a high storage capacity contains mobile water in the fissures and stagnant water in the porous matrix. The water enters this system at the surface and flows through it to the drained channels which are regarded as a separate flow system finally draining to both springs. The channels are connected with sinkholes which introduce additional water directly from the surface. The conceptual model of water flow in this basin is shown in Fig. 4.

It was assumed that for baseflow there is no flow in the drainage channels. Then, tritium sampled in the springs represented the flow through the fissured-porous aquifer. During high flow rates, the seasonal variations in $^{18}$O were assumed to represent the fast flow through the channels. Fitting of the dispersion model to the tritium data (Fig. 5) and the piston flow model to the $^{18}$O data, yielded the mean transit times and volumes of water in both flow systems, and additionally, the portion of water flowing directly from the sinkholes through the drainage channels. The volume of water stored in the karstic channels ($V_c$) was found to be insignificantly small (less
than 1%) in comparison to the volume of water stored in the fissured-porous aquifer ($V_p$). On the other hand through the karstic channels the whole volumetric water flux ($Q$) flows. The volumetric flow rate of water entering the channels through the sinkholes ($Q_s$) was relatively large (17.5% of total discharge). This water flux, having a very short transit time to the springs (about one month), constitutes a potential threat for the water quality in the springs in the case of possible contamination at the surface of the catchment. The resulting hydrological parameters are shown in Fig. 4.

Similar tracer studies are presented in e.g. Maloszewski et al. (1992).

REFERENCES


