Calibrating the SEDD model for Sicilian ungauged basins

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Abstract Identifying the areas in a drainage basin that are the most sensitive or susceptible to erosion stimulated the study of within-basin variability of the sediment delivery processes and the use of spatially distributed models coupled with Geographical Information Systems. In this paper, the SEdiment Delivery Distributed (SEDD) model applicable at the morphological unit scale is reviewed. The model couples the Universal Soil Loss Equation (USLE), in which different expressions of the topographic factors are considered, with a relationship for evaluating the sediment delivery ratio of each morphological unit. The model is applied to six Sicilian drainage basins, each having a reservoir at the outlet, with areas ranging from 20 to 70 km². For each drainage basin, the model is applied using a raster scheme and a subroutine of Arc-Info software for identifying the hydraulic path linking each hillslope cell to the nearest stream cell. Finally, a procedure for estimating the single coefficient of the SEDD model for ungauged basins is proposed.

Key words distributed models; GIS; SEDD model; sediment yield measurements; soil erosion; USLE

INTRODUCTION

Each year, the world is losing $23 \times 10^9$ t of soil from cropland in excess of new soil formation (Walling & Quine, 1992). Accelerated soil erosion is therefore a serious problem, having environmental effects such as reservoir sedimentation and nonpoint source pollution. Conservation planning needs an optimal allocation of soil conservation and land management practices within the drainage basin, and has stimulated the use of spatially distributed sediment yield models coupled with GIS. Soil erosion and sediment yield models are helpful for the land-use planner who must decide if a specified design or land management practice allows meeting a specific soil loss tolerance goal.

At the drainage basin scale, predicting the sediment yield (i.e. the quantity of sediment that is transferred in a given time interval from the sediment sources through the channel network to the basin outlet) can be carried out by coupling a soil erosion model with a mathematical function expressing the sediment transport efficiency of the hillslopes and the channel network (Walling, 1983; Bagarello et al., 1991). If the process is studied at the mean annual temporal scale, the sediment transport efficiency of the hillslopes and the channel network is usually represented by the spatially...
lumped concept of the basin sediment delivery ratio $SDR_w$. According to Boyce (1975), the sediment delivery ratio $SDR_w$ generally decreases with increasing basin size. At the mean annual temporal scale, Richards (1993) suggested that, according to Playfair's law of stream morphology, over a long time a stream must transport essentially all sediment delivered to it from the hillslopes. Richards (1993) also suggested that sediment is produced by different sources distributed throughout the drainage basin, and that sediment delivery processes should be modelled by employing a spatially distributed approach considering individual fields. In other words, the drainage basin is divided into individual units having a regular (square or triangular cells) or irregular shape (morphological units) (Bagarello et al., 1993). In each unit, the variables appearing in the equations of a process-oriented model or the factors of a parametric model such as Universal Soil Loss Equation (USLE) can be calculated. The existing difficulties of physically-based modelling (e.g. numerous input parameters and uncertainties of the selected equations, see Beven, 1989) increase the attractiveness of a parametric soil erosion model coupled with a spatial disaggregation criterion for the sediment delivery process.

For a drainage basin discretized into morphological units (i.e. areas of clearly defined aspect, length and steepness) (Bagarello et al., 1993), Ferro & Minacapilli (1995) proposed to model the within-basin variability of the hillslope sediment delivery processes using a sequential approach. Basically, this approach follows the sediment mass in a Lagrangian scheme, and applies appropriate sediment delivery factors $SDR_i$ to each sequential morphological unit (Novotny & Chesters, 1989).

The experimental testing of the distributed models is generally carried out by comparing, at the selected temporal scale, the calculated and the measured sediment yield at the drainage basin outlet. The internal functioning of the SEdiment Delivery Distributed (SEDD) model, i.e. its ability to calculate the spatial distribution of the sediment yield, was tested with positive results at the mean annual temporal scale using measurements of caesium-137 activity of soil samples (Ferro et al., 1998; Di Stefano et al., 2000b). Most applications of the SEDD model were carried out using an approach that divides the drainage basin into morphological units with irregular shapes that are manually delineated on topographic maps. Using a GIS introduced the raster scheme that divides the study area into square cells with a known grid size $D$ and the numerical and alphanumeric information stored in a matrix scheme. The raster scheme allows the objective application of the SEDD model and reduces the calculation time.

In this paper, the theoretical basis and the analytical formulation of SEDD model (Ferro & Porto, 2000) are briefly reviewed. The model will be applied to six Sicilian drainage basins, each having a reservoir at the outlet, with areas ranging from 20 to 70 km$^2$. For each drainage basin, the model is applied using a raster scheme and a subroutine of ArcInfo software to identify the hydraulic path linking each hillslope square cell to the nearest stream cell. Finally, a procedure for estimating the single parameter of the SEDD model for ungauged basins will be proposed.

**A REVIEW OF THE SEDIMENT DELIVERY DISTRIBUTED MODEL**

For a drainage basin divided into morphological units, and neglecting the channel component of the sediment delivery processes, Ferro & Minacapilli (1995) proposed to
calculate the sediment delivery ratio $SDR_i$ of each morphological unit by the following relationship:

$$SDR_i = \exp(-\beta t_{p,i}) = \exp\left(-\frac{\beta l_{p,i}}{s_{p,i}}\right) = \exp\left(-\beta \sum_{j=1}^{N_p} \frac{\lambda_{i,j}}{s_{i,j}}\right)$$  \hspace{1cm} (1)$$

in which $\beta$ is a coefficient $[m^{-1}]$, $t_{p,i}$ is the travel time, expressed as $[m]$, of the particles eroded from a given hillslope cell to the nearest stream cell, $N_p$ is the number of hillslope cells localized along the hydraulic path having a length $l_{p,i}$ and a slope $s_{p,i}$, and $\lambda_{i,j}$ and $s_{i,j}$ are the length and slope, respectively, of each hillslope cell $i$ located along the hydraulic path $j$.

The sediment production of each hillslope cell $Y_i$, expressed as metric tonnes, is calculated by the following equation:

$$Y_i = R_i K_i LS_i C_i P_i SDR_i S_{u,i}$$  \hspace{1cm} (2)$$

in which $R_i$ is the rainfall erosivity factor of the $i$th cell $[t \cdot ha^{-1} \cdot unit \ of \ A; t \cdot unit \ of \ km^2]$, $K_i$ is the soil erodibility factor $[t \cdot h \cdot kg^{-1} \cdot m^2]$ estimated by the procedure of Wischmeier et al. (1971), $C_i$ is the crop factor, $P_i$ is the conservation practice factor, $LS_i$ is the topographic factor and $S_{u,i}$ is the area of each hillslope cell. Three expressions are used for calculating the topographic factor. For the original USLE model (Wischmeier & Smith, 1978):

$$LS_i = \left(\frac{\lambda_{i,j}}{22.1}\right)^{0.5} \left(\frac{0.43 + 0.30 s_i + 0.043 s_i^2}{0.0896}\right)$$  \hspace{1cm} (3)$$

in which $s_i$ is the slope (%) of the hillslope cell. McCool et al. (1989) proposed:

$$LS_i = \left(\frac{\lambda_{i,j}}{22.13}\right)^{m_i} \left(10.8 \sin \alpha_i + 0.03\right) \text{ if } \tan \alpha_i < 0.09$$ \hspace{1cm} (4a)$$

$$LS_i = \left(\frac{\lambda_{i,j}}{22.13}\right)^{m_i} \left(16.8 \sin \alpha_i - 0.5\right) \text{ if } \tan \alpha_i \geq 0.09$$ \hspace{1cm} (4b)$$

In Equation (4), the slope length exponent $m_i$ is calculated as:

$$m_i = \frac{a f_i}{1 + a f_i}$$  \hspace{1cm} (5)$$

in which, according to McCool et al. (1989), Di Stefano et al. (2000a), the coefficient $a$ is set equal to one and $f_i$ has the following expression:

$$f_i = \frac{\sin \alpha_i}{0.0896 (3 \sin 0.8 \alpha_i + 0.56)}$$  \hspace{1cm} (6)$$

Moore & Burch (1986) proposed:

$$LS_i = \left(\frac{\lambda_{i,j}}{22.13}\right)^{0.6} \left(\frac{\sin \alpha_i}{0.0896}\right)^{1.3}$$  \hspace{1cm} (7)$$

For evaluating the coefficient $\beta$ in equation (1), the sediment balance equation at drainage basin outlet is applied. This equation assumes that the sediment yield $Y_s(t)$ is
calculated by summing the sediment yield values $Y_i$ of all morphological units (square cells) within the basin:

$$Y_s = \sum_{i=1}^{N_h} SDR_i R_i K_i LS_i C_i P_i S_{u,i} = \sum_{i=1}^{N_h} \exp\left( -\beta t_{p,i} \right) R_i K_i LS_i C_i P_i S_{u,i}$$

(8)

where $N_h$ is the number of hillslope cells in the basin.

The sediment balance equation at the basin outlet allows determination of the sediment delivery relationship between $SDR_w$ and the sediment delivery ratio $SDR_i$ of each hillslope cell. Ferro & Minacapilli (1995) showed that the sediment delivery relationship is independent of the selected soil erosion model, and can be expressed using morphological data only:

$$SDR_w = \frac{\sum_{i=1}^{N_h} \exp\left( -\beta t_{p,i} \right) \lambda_{i,j}^{0.5} s_i^2 S_{u,i}}{\sum_{i=1}^{N_h} \lambda_{i,j}^{0.5} s_i^2 S_{u,i}}$$

(9)

STUDY AREA AND RESEARCH METHODS

The SEDD model was applied to six Sicilian drainage basins, each having a reservoir located at the basin outlet, with areas ranging from 20 to 70 km$^2$. For each drainage basin, the sedimentation volume of the reservoir over a known time interval of several years is available (Fig. 1). Table 1 lists for each basin the drainage basin area $S_w$ (km$^2$), the mean hillslope slope $s$ (%), the mean altitude $H$ (m), and the mean annual sedimentation rate $I$ (t ha$^{-1}$ year$^{-1}$) (Tamburino et al., 1989; La Loggia & Minacapilli, 1998). A digital terrain model was obtained using a topographic map at scale of 1:25 000 and a raster scheme with a grid size $D$ of 50 m. For calculating the topographic factor $LS_i$ of each hillslope cell and the travel time $t_{p,i}$, the slope and aspect of each cell were calculated (Burrough, 1996) and the stream network was extracted (Jenson & Domingue, 1988).
Table 1 Characteristic data of the investigated Sicilian drainage basins.

<table>
<thead>
<tr>
<th>Basin</th>
<th>$S_w$ (km$^2$)</th>
<th>$s$ (%)</th>
<th>$H$ (m)</th>
<th>$I$ (t ha$^{-1}$ year$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prizzi</td>
<td>19</td>
<td>17</td>
<td>770</td>
<td>13.9</td>
</tr>
<tr>
<td>Piana degli Albanesi</td>
<td>35</td>
<td>18</td>
<td>759</td>
<td>18.7</td>
</tr>
<tr>
<td>Rubino</td>
<td>40</td>
<td>13</td>
<td>315</td>
<td>2.7</td>
</tr>
<tr>
<td>Nicoletti</td>
<td>49</td>
<td>18</td>
<td>599</td>
<td>4.7</td>
</tr>
<tr>
<td>Ancipa</td>
<td>49</td>
<td>20</td>
<td>1245</td>
<td>7.4</td>
</tr>
<tr>
<td>Gammauta</td>
<td>69</td>
<td>28</td>
<td>820</td>
<td>6.9</td>
</tr>
</tbody>
</table>

The topographic factor (equations 3, 4 and 7) and the travel time $t_{p,i}$ into each cell were calculated using a slope length taking into account the aspect of the cell ($D$ or $1.41 D$, depending on the aspect). The hillslope travel time from a source cell to a stream cell (Fig. 2) was obtained by summing the travel times $t_{p,i}$ of all cells localized along the hillslope hydraulic path from the source to the stream cell. As an example, Fig. 3 shows the hydraulic path automatically plotted using the ArcInfo functions “spatial modelling” and “map algebra.” In the same figure, the DEM and the extracted stream network are plotted. Figure 4 shows an example of the spatial distribution of travel time, expressed as (m), obtained by the procedure.

The rainfall erosivity factor $R$ was obtained by digitizing the Sicilian iso-erosivity map (Ferro et al., 1991). The soil erodibility factor $K$ was calculated using the nomograph of Wischmeier et al. (1971) and soil samples collected in the soil mapping units of each basin. The crop factor $C$ was estimated using information from Wischmeier & Smith (1978) and De Tar et al. (1980). Soil conservation measures were not carried out in the investigated drainage basins and therefore the conservation practice factor $P$ was assumed to be equal to 1. For each investigated drainage basin, a Landsat TM image was used to obtain a land-use map having a mesh size comparable with the one used for the DEM.

![Fig. 2 Scheme for calculating the travel time from a source cell to a stream cell.](image-url)
RESULTS

For each investigated drainage basin, the basin soil erosion rate $A_s$ (t ha$^{-1}$ year$^{-1}$) and its spatial distribution were calculated by applying the SEDD model. Using the measured sediment yield values $I$ (t ha$^{-1}$ year$^{-1}$), the basin sediment delivery ratio $SDR_w$ equal to the ratio $I/A_s$ was also calculated.
Calibrating the SEDD model for Sicilian ungauged basins

Table 2 Results of applying the SEDD model.

<table>
<thead>
<tr>
<th>Basin</th>
<th>Prizzi</th>
<th>Piana degli Albanesi</th>
<th>Rubino</th>
<th>Nicoletti</th>
<th>Ancipa</th>
<th>Gammaruta</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>45</td>
<td>45.2</td>
<td>30</td>
<td>80</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$K$</td>
<td>0.442</td>
<td>0.512</td>
<td>0.489</td>
<td>0.532</td>
<td>0.544</td>
<td>0.431</td>
</tr>
<tr>
<td>$LS$ (USLE)</td>
<td>5.3</td>
<td>9.3</td>
<td>4.2</td>
<td>5.4</td>
<td>6.6</td>
<td>13.38</td>
</tr>
<tr>
<td>$LS$ (RUSLE)</td>
<td>3.8</td>
<td>5.2</td>
<td>3.1</td>
<td>4.4</td>
<td>5.3</td>
<td>7.23</td>
</tr>
<tr>
<td>$LS$ (M&amp;B)</td>
<td>3.5</td>
<td>4.6</td>
<td>2.8</td>
<td>3.9</td>
<td>4.7</td>
<td>6.68</td>
</tr>
<tr>
<td>$C$</td>
<td>0.401</td>
<td>0.392</td>
<td>0.243</td>
<td>0.208</td>
<td>0.337</td>
<td>0.329</td>
</tr>
<tr>
<td>$A$ (USLE) (t ha$^{-1}$ year$^{-1}$)</td>
<td>38.7</td>
<td>74.6</td>
<td>10.8</td>
<td>34.2</td>
<td>58.3</td>
<td>72.7</td>
</tr>
<tr>
<td>$A$ (RUSLE) (t ha$^{-1}$ year$^{-1}$)</td>
<td>28.1</td>
<td>41.7</td>
<td>8.6</td>
<td>34.4</td>
<td>47.2</td>
<td>43.82</td>
</tr>
<tr>
<td>$A$ (M&amp;B) (t ha$^{-1}$ year$^{-1}$)</td>
<td>25.5</td>
<td>38.4</td>
<td>7.8</td>
<td>30.7</td>
<td>41.4</td>
<td>40.16</td>
</tr>
<tr>
<td>$\beta$ (USLE)</td>
<td>0.0015</td>
<td>0.0013</td>
<td>0.0018</td>
<td>0.0055</td>
<td>0.0060</td>
<td>0.0093</td>
</tr>
<tr>
<td>$\beta$ (RUSLE)</td>
<td>0.0008</td>
<td>0.0007</td>
<td>0.0014</td>
<td>0.0057</td>
<td>0.0049</td>
<td>0.0054</td>
</tr>
<tr>
<td>$\beta$ (M&amp;B)</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0013</td>
<td>0.0051</td>
<td>0.0044</td>
<td>0.0051</td>
</tr>
<tr>
<td>$SDR_w$ (USLE)</td>
<td>0.359</td>
<td>0.251</td>
<td>0.234</td>
<td>0.139</td>
<td>0.128</td>
<td>0.095</td>
</tr>
<tr>
<td>$SDR_w$ (RUSLE)</td>
<td>0.495</td>
<td>0.432</td>
<td>0.292</td>
<td>0.138</td>
<td>0.158</td>
<td>0.157</td>
</tr>
<tr>
<td>$SDR_w$ (M&amp;B)</td>
<td>0.545</td>
<td>0.487</td>
<td>0.324</td>
<td>0.155</td>
<td>0.181</td>
<td>0.172</td>
</tr>
<tr>
<td>$\beta_s$ (USLE)</td>
<td>0.0009</td>
<td>0.0011</td>
<td>0.0020</td>
<td>0.0047</td>
<td>0.0050</td>
<td>0.0108</td>
</tr>
<tr>
<td>$\beta_s$ (RUSLE)</td>
<td>0.0007</td>
<td>0.0009</td>
<td>0.0015</td>
<td>0.0034</td>
<td>0.0036</td>
<td>0.0075</td>
</tr>
<tr>
<td>$\beta_s$ (M&amp;B)</td>
<td>0.0007</td>
<td>0.0008</td>
<td>0.0014</td>
<td>0.0030</td>
<td>0.0032</td>
<td>0.0065</td>
</tr>
</tbody>
</table>

For each basin, the values of $\beta$ corresponding to the condition $Y_s = I$ were calculated applying the sediment balance equation (8). Table 2 lists the results obtained, and shows for each basin the mean values of the USLE's factors ($R$, $K$, $LS$, $C$), the mean annual soil loss $A$ (t ha$^{-1}$ year$^{-1}$), the values of $\beta$ and the $SDR_w$ values calculated as the ratio between the measured sediment yield $I$ and the annual soil loss $A$.

For the six investigated basins, the correlation between the sediment delivery ratio $SDR_w$ and morphometric variables representing the hillslope sediment transport efficiency was investigated. Figure 5 shows that the sediment delivery ratio is strongly correlated with drainage area $S$. The relationship is described by the following equation:

$$SDR_w = \exp (- b S) \quad (10)$$

in which $S$ is expressed in km$^2$ and $b$ is a numerical coefficient assuming a value quasi-independent of the expression used for the topographic factor ($b = 0.0392$ for USLE, $b = 0.0328$ for RUSLE and $b = 0.0306$ for the expression of Moore & Burch). This result is consistent with previous studies (ASCE, 1975; Bagarello et al., 1991).

For an ungauged basin, applying equation (10) provides an estimate of the basin sediment delivery ratio, and from equation (9) an estimate, $\beta_s$, of $\beta$ can be obtained. Applying equation (9) also requires knowledge of the slope length and slope steepness of each hillslope cell of the digital elevation model. Table 2 shows that the estimates $\beta_s$, corresponding to different expressions of the topographic factors, are very near to the $\beta$ values calculated by the sediment balance equation with $Y_s = I$. Finally, using $\beta_s$, equation (8) was applied to calculate the basin sediment yields and the corresponding...
Fig. 5 Relationship between sediment delivery ratio and basin area.

spatial distributions (Fig. 6). Figure 7 shows a good agreement between the sediment yield values calculated by equation (8) with \( \beta = \beta_s \) and the measured sediment yield \( I \). This agreement confirms the reliability of the proposed procedure (equations 9 and 10) for estimating the coefficient \( \beta \), and makes the application of the SEDD model to ungauged basins possible.

**CONCLUSIONS**

Recent studies show that sediment sources are not necessarily the within-basin areas interested to high erosion phenomena because of the reduction due to the ability of hillslopes and stream network to transport sediment particles. For identifying areas within a drainage basin in which soil conservation strategies are needed, a soil erosion model coupled with a mathematical function expressing the sediment delivery processes has to be applied.
Fig. 6 Example of the spatial distribution of sediment yield (t ha\(^{-1}\) year\(^{-1}\)).

Fig. 7 Comparison between measured and calculated sediment yield.
In this paper, the theoretical bases and the analytical formulation of the SEDD (SEDiment Delivery Distributed) were reviewed first. The model has already been tested at different spatial (from 4 ha to 100 km²) and temporal (event, annual, mean annual) scales, and the spatial distribution of the sediment yield within the basin was also tested by the caesium-137 technique. Then the model was applied to six Sicilian drainage basins, with areas ranging from 20 to 70 km², each having both a reservoir at the outlet and a measurement of the sedimentation volume over a known time interval of several years. For each drainage basin the model was applied using a raster scheme and a subroutine of the ArcInfo software identifying the hydraulic path linking each hillslope cell to the nearest stream cell. For each of the investigated drainage basins, using the sediment balance equation at the outlet drainage basin the estimate of the β coefficient of the SEDD model was obtained.

Finally, using equation (10) for estimating the basin sediment delivery ratio and the morphological relationship, equation (9), a procedure for estimating the coefficient β for ungauged basins was established. For the six drainage basins investigated, the agreement between the measured values of sediment yield and those calculated by the sediment balance equation and the procedure for estimating β provided a positive test of the accuracy of this procedure.

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